Variation

by : rraezzz

1.1 DIRECT VARIATION

- Explains the relation between two variables such that when variable y increase, then variable x also increase at the same rate and vice versa
- This relation can be written as y varies directly as x

EXAMPLE

The total wage of a part time worker as a promoter varies directly as the number of hours he works. State the change on

a) The total wage if the number of working hours is double
b) The total wage if the number of working hours decreases by 50%
c) The number of working hours if the total wage is halved

SOLUTION

a) The total wage is doubled
b) The total wage decreases by 50%
c) The number of working hours is halved

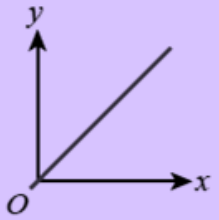
RELATIONSHIP BETWEEN TWO VARIABLES FOR A DIRECT VARIATION

The results of Mind Mobilisation 2 show that for a direct variation,

(a) the value of $\frac{y}{x}$ is a constant,

(b) the graph of y against x is a straight line that passes through the origin,

(c) the variable y varies directly as the variable x.



Bambi

Variation

by : rraezzz

For a **direct variation**, y varies directly as x can be written as

$$y \propto x \quad \text{(variation relation)}$$
$$y = kx \quad \text{(equation form)}$$

where k is a constant.

Relation between the value of k and the concept of proportion:

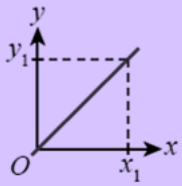
y	y_1	y_2	y_3	y_4
x	x_1	x_2	x_3	x_4

$$k = \frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \frac{y_4}{x_4}$$

Relation between the value of k and the gradient of a straight line passes through the origin:

Gradient,

$$m = \frac{y_1}{x_1}$$
$$= k$$



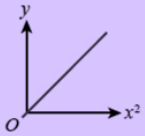
This relation is true only if the straight line passes through the origin.

The results of Mind Mobilisation 3 show that for a direct variation,

(a) the value of $\frac{y}{x^2}$ is a constant. Hence, $k = \frac{y}{x^2}$,

(b) the graph of y against x^2 is a straight line that passes through the origin,

(c) variable y varies directly as variable x^2.



In general,

For a **direct variation**, y varies directly as x^n can be written as

$y \propto x^n$ (variation relation)

$y = kx^n$ (equation form)

where $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and k is a constant.

The graph of y against x^n is a straight line passes through the origin where k is the gradient of the straight line.

Critical Mind

In Mind Mobilisation 3, what is the relation between the values of π and $\frac{y}{x^2}$?

Smart TiPS

$y \propto x$ is true if and only if $n = 1$.

Bambi

Variation

by : rraezzz

Example 2

A toy car moves from a state of rest. The distance travelled by the toy car, y, changes with time, t, as shown in the table below.

Time, t (s)	2	4	6	8	10	12
Distance, y (cm)	14	28	42	56	70	84

Interactive Platform

When $y = 3x^2$, y does not vary directly as x. Discuss.

Determine whether y varies directly as t or t^2. Hence, write the relation in the form of variation.

Solution:

t	2	4	6	8	10	12
y	14	28	42	56	70	84
$\frac{y}{t}$	$\frac{14}{2} = 7$	$\frac{28}{4} = 7$	$\frac{42}{6} = 7$	$\frac{56}{8} = 7$	$\frac{70}{10} = 7$	$\frac{84}{12} = 7$
$\frac{y}{t^2}$	$\frac{14}{2^2} = 3.50$	$\frac{28}{4^2} = 1.75$	$\frac{42}{6^2} = 1.17$	$\frac{56}{8^2} = 0.88$	$\frac{70}{10^2} = 0.70$	$\frac{84}{12^2} = 0.58$

y varies directly as t because the value of $\frac{y}{t}$ is a constant. Hence, $y \propto t$.

y does not vary directly as t^2 because $\frac{y}{t^2}$ is not a constant.

Example 3

(a) By drawing the graph of y against x, determine whether y varies directly as x.

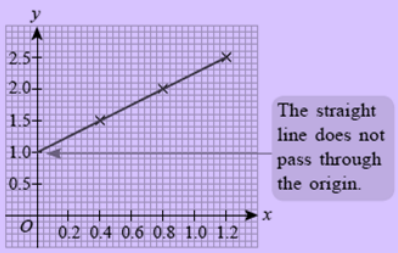
x	0.4	0.8	1.2
y	1.5	2.0	2.5

(b) By drawing the graph of y against x^2, determine whether y varies directly as x^2.

x	2	3	4	5
y	8	18	32	50

Solution:

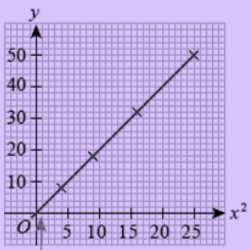
(a)



The straight line does not pass through the origin.

Hence, y does not vary directly as x.

(b)



x^2	y
4	8
9	18
16	32
25	50

The straight line passes through the origin.

Hence, y varies directly as x^2.

Bambi

Variation

by : rraezzz

Example 4

It is given $m = 0.8$ when $n = 0.125$. Express m in terms of n if
(a) m varies directly as n,
(b) m varies directly as the cube root of n.

Solution:

(a) $m \propto n$

$m = kn$ ← Write the relation in the form of equation.

$0.8 = k(0.125)$ ← Substitute the values of m and n into the equation to obtain the value of k.

$k = \dfrac{0.8}{0.125}$

$= 6.4$

Hence, $m = 6.4n$

(b) $m \propto \sqrt[3]{n}$

$m = k\sqrt[3]{n}$

$0.8 = k(\sqrt[3]{0.125})$

$k = \dfrac{0.8}{\sqrt[3]{0.125}}$

$= 1.6$

Hence, $m = 1.6\sqrt[3]{n}$

Example 5

The extension of a spring, x cm, varies directly as the mass of the load, w g, hanging on the string. It is given the extension of a spring is 3 cm when a load of 200 g is hung on the string. Express x in terms of w.

Solution:

$x \propto w$

$x = kw$ ← Write the relation in the form of equation.

$3 = k(200)$ ← Substitute the values of x and w into the equation to obtain the value of k.

$k = \dfrac{3}{200}$

$= 0.015$

Hence, $x = 0.015w$

Bambi

Variation

by : rraezzz

For a **joint variation**, y varies jointly as x^m and z^n can be written as

$y \propto x^m z^n$ (variation relation)

$y = kx^m z^n$ (equation form)

where

$m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$,

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and

k is a constant.

Example 9

Write the relation using the symbol $\propto$ and in equation form for each of the following.

(a) p varies directly as q and $\sqrt{r}$.

(b) y varies directly as the square of w and the cube of x.

(c) The volume of a prism, V varies directly as the area of cross-section, A and its height, h.

(d) The mass, w of a cylindrical metal rod varies directly as its length, p and the square of the diameter of its base, d.

Smart TiPS

$y \propto xz$ can be read as
- y varies directly as x and z
- y varies jointly as x and z
- y is jointly proportional to x and z

Solution:

(a) $p \propto q\sqrt{r}$ ← $p \propto q$ and $p \propto \sqrt{r}$

$p = kq\sqrt{r}$

(b) $y \propto w^2x^3$ ← $y \propto w^2$ and $y \propto x^3$

$y = kw^2x^3$

(c) $V \propto Ah$ ← $V \propto A$ and $V \propto h$

$V = kAh$

(d) $w \propto pd^2$ ← $w \propto p$ and $w \propto d^2$

$w = kpd^2$

Example 10

Given that $x \propto y^2z$, express x in terms of y and z if $x = 6$ when $y = 3$ and $z = 5$.

Solution:

$x \propto y^2z$

$x = ky^2z$ ← Write the relation in the form of equation.

$6 = k(3)^2(5)$ ← Substitute the values of x, y and z into the equation to obtain the value of k.

$k = \frac{6}{(3)^2(5)}$

$= \frac{2}{15}$

Hence, $x = \frac{2}{15}y^2z$

Bambi

Variation

by : rraezzz

1.2 INVERSE VARIATION

- In inverse variation, variable y increases when the variable x decreases at the same rate and vice versa.
- This relation can be written as y varies inversely as x.

EXAMPLE

Example 15

Chia Ming takes part in a 42-km marathon. The table below shows the relation between the time taken by Chia Ming and his average speed.

Time, t (hours)	4	5	6	7	8
Average speed, v (km/h)	10.50	8.40	7.00	6.00	5.25

State the change in average speed if the time taken
(a) increases by two times,
(b) decreases by 1.5 times.

Solution:

(a)

t	v
4	10.50
8	5.25

$\times 2$ ... $\div 2$

When the time increases two times, then the average speed decreases two times.

(b)

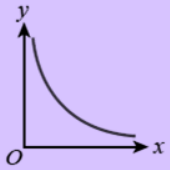
t	v
6	7.00
4	10.50

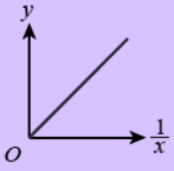
$\div 1.5$ ... $\times 1.5$

When the time decreases 1.5 times, then the average speed increases 1.5 times.

RELATIONSHIP BETWEEN TWO VARIABLES FOR A INVERSE VARIATION

The results of Mind Mobilisation 6 show that for an inverse variation,
(a) the value of xy is a constant. Hence, $k = xy$,
(b) the graph of y against x is a hyperbola and the graph of y against $\frac{1}{x}$ is a straight line that starts from the origin,
(c) y varies inversely as x.





Bambi

Variation

by : rraezzz

Relation between the value of k and the concept of proportion:

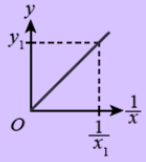
y	y_1	y_2	y_3	y_4
x	x_1	x_2	x_3	x_4

$k = x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4$

Relation between the value of k and the gradient of the straight line that starts from the origin for the graph of y against $\frac{1}{x}$:

Gradient,

$$m = \frac{y_1}{\frac{1}{x_1}}$$
$$= x_1y_1$$
$$= k$$



This relation is true only if the straight line starts from the origin.

Smart TiPS

The relation of inverse variation written as 'y varies inversely as x' can also be written as 'x and y are inversely proportional'.

Hence, we can relate the two variables x and y in the form of equation, that is

$$xy = k$$
$$y = \frac{k}{x}.$$

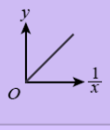
For an **inverse variation**, y varies inversely as x can be written as

$$y \propto \frac{1}{x} \text{ (variation relation)}$$
$$y = \frac{k}{x} \text{ (equation form)}$$

where k is a constant.

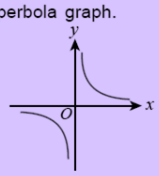
(a) The graph of y against x is a hyperbola.

(b) The graph of y against $\frac{1}{x}$ is a straight line that starts from the origin ($x \neq 0$).



MEMORY BOX

$y = \frac{a}{x}$, $x \neq 0$ is a reciprocal function with a hyperbola graph.

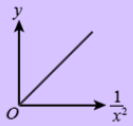


The results of Mind Mobilisation 7 show that for an inverse variation,

(a) the value of x^2y is a constant. Hence, $k = x^2y$,

(b) the graph of y against $\frac{1}{x^2}$ is a straight line that starts from the origin,

(c) y varies inversely as x^2.



In general,

For an **inverse variation**, y varies inversely as x^n can be written as

$$y \propto \frac{1}{x^n} \text{ (variation relation)}$$
$$y = \frac{k}{x^n} \text{ (equation form)}$$

where $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and k is a constant.

The graph of y against $\frac{1}{x^n}$ is a straight line that starts from the origin ($x \neq 0$) where k is the gradient of the straight line.

Interactive Platform

When $y = \frac{1}{x^2}$, y does not vary directly as x. Discuss this statement.

Smart TiPS

$y \propto \frac{1}{x}$ is true if and only if $n = 1$.

Bambi

Variation

by : rraezzz

Example 16

It is given that the time needed, t, to assemble a piece of furniture varies inversely as the number of workers, x. The table below shows the relation between x and t.

Number of workers, x	2	3	4	5	6
Time needed, t (minutes)	180	120	90	72	60

By calculating the values of xt and x^2t, determine whether t varies inversely as x or x^2. Hence, write the relation by using the symbol $\propto$.

Solution:

x	2	3	4	5	6
t	180	120	90	72	60
xt	$2(180) = 360$	$3(120) = 360$	$4(90) = 360$	$5(72) = 360$	$6(60) = 360$
x^2t	$2^2(180) = 720$	$3^2(120) = 1080$	$4^2(90) = 1440$	$5^2(72) = 1800$	$6^2(60) = 2160$

The value of xt is a constant, whereas the value of x^2t is not a constant. Hence, t varies inversely as x, that is $t \propto \frac{1}{x}$.

Example 20

It is given that p varies inversely as q. If $p = 2$ when $q = 7$, calculate the value of p when $q = 1.6$.

Solution:

$p \propto \frac{1}{q}$

$p = \frac{k}{q}$

$2 = \frac{k}{7}$

$k = 2(7)$

$= 14$

Hence, $p = \frac{14}{q}$

When $q = 1.6$,

$p = \frac{14}{1.6}$

$= 8.75$

Alternative Method:

Using the concept of proportion:
Given that $p_1 = 2$, $q_1 = 7$, $q_2 = 1.6$

$p_1 q_1 = p_2 q_2$

$2 \times 7 = p_2 \times 1.6$

$p_2 = \frac{2 \times 7}{1.6}$

$= 8.75$

Example 18

It is given that $x = 0.25$ when $y = 3$. Express y in terms of x if
(a) y varies inversely as x,
(b) y varies inversely as the square root of x.

Solution:

(a) $y \propto \frac{1}{x}$

$y = \frac{k}{x}$ ← Write the relation in the form of equation.

$3 = \frac{k}{0.25}$ ← Substitute the values of y and x into the equation to obtain the value of k.

$k = 3(0.25)$

$= 0.75$

Hence, $y = \frac{0.75}{x}$

(b) $y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$3 = \frac{k}{\sqrt{0.25}}$

$k = 3(\sqrt{0.25})$

$= 1.5$

Hence, $y = \frac{1.5}{\sqrt{x}}$

Smart TiPS

(i) y varies inversely as x,
$y \propto \frac{1}{x}$

(ii) y varies inversely as the square of x,
$y \propto \frac{1}{x^2}$

(iii) y varies inversely as the cube of x,
$y \propto \frac{1}{x^3}$

(iv) y varies inversely as the square root of x,
$y \propto \frac{1}{\sqrt{x}}$

(v) y varies inversely as the cube root of x,
$y \propto \frac{1}{\sqrt[3]{x}}$

Bambi

Variation

by : rraezzz

1.3 COMBINED VARIATION

❖ **Combined variation involves a combination of direct variation or joint variation, and inverse variation**

The results of Mind Mobilisation 8 show that y varies directly as x, and y varies inversely as z. Therefore, y varies directly as x and inversely as z, that is $y \propto \frac{x}{z}$.

In general,

For a **combined variation**, y varies directly as x^m and inversely as z^n can be written as

$y \propto \frac{x^m}{z^n}$ (variation relation)

$y = \frac{kx^m}{z^n}$ (equation form)

where

$m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$, $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and

k is a constant.

Example 22

It is given that y varies directly as the square of x and inversely as the square root of z. If $y = 8$ when $x = 4$ and $z = 36$, express y in terms of x and z.

Solution:

$$y \propto \frac{x^2}{\sqrt{z}}$$

$$y = \frac{kx^2}{\sqrt{z}}$$ ← Write the relation between y, x and z in the form of equation.

$$8 = \frac{k(4)^2}{\sqrt{36}}$$ ← Substitute the values of y, x and z into the equation to obtain the value of k.

$$k = \frac{8\sqrt{36}}{4^2}$$

$$= 3$$

Hence, $y = \frac{3x^2}{\sqrt{z}}$

Smart TiPS

(i) y varies directly as w and the cube of x, and inversely as the cube root of z,

$$y \propto \frac{wx^3}{\sqrt[3]{z}}$$

(ii) y varies directly as the product of w^2 and $\sqrt{x}$, and inversely as z^3,

$$y \propto \frac{w^2\sqrt{x}}{z^3}$$

Bambi

Variation

by : rraezzz

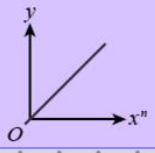
CONCLUSION

Direct Variation

y varies directly as x^n, for

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

- $y \propto x^n$
- $y = kx^n$ where k is a constant and $k = \frac{y}{x^n}$
- The graph of y against x^n is a straight line that passes through the origin



Joint Variation

A direct variation where a variable varies as the product of two or more variables.

- y varies jointly as x^m and z^n, for

  $m = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and

  $n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

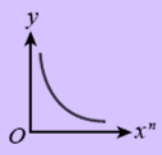
- $y \propto x^m z^n$
- $y = kx^m z^n$ where k is a constant and $k = \frac{y}{x^m z^n}$

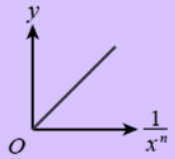
Inverse Variation

y varies inversely as x^n, for

$n = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

- $y \propto \frac{1}{x^n}$
- $y = \frac{k}{x^n}$ where k is a constant and $k = yx^n$
- The graph of y against $\frac{1}{x^n}$ is a straight line that starts from the origin ($x \neq 0$).





y varies inversely as x^n

Combined Variation

Involves a combination of direct variation or joint variation, and inverse variation.

- For $p = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$, $q = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$ and $r = 1, 2, 3, \frac{1}{2}, \frac{1}{3}$

(i) $y \propto \frac{x^p}{z^q}$

$y = \frac{kx^p}{z^q}$

where k is a constant.

(ii) $y \propto \frac{w^p x^q}{z^r}$

$y = \frac{kw^p x^q}{z^r}$